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Waterways Experiment  
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# Reliability Analysis of a Reinforced Concrete Drainage Structure

*by Robert C. Patev, Mary Ann Leggett*

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Prepared for U.S. Army Engineer District, Vicksburg

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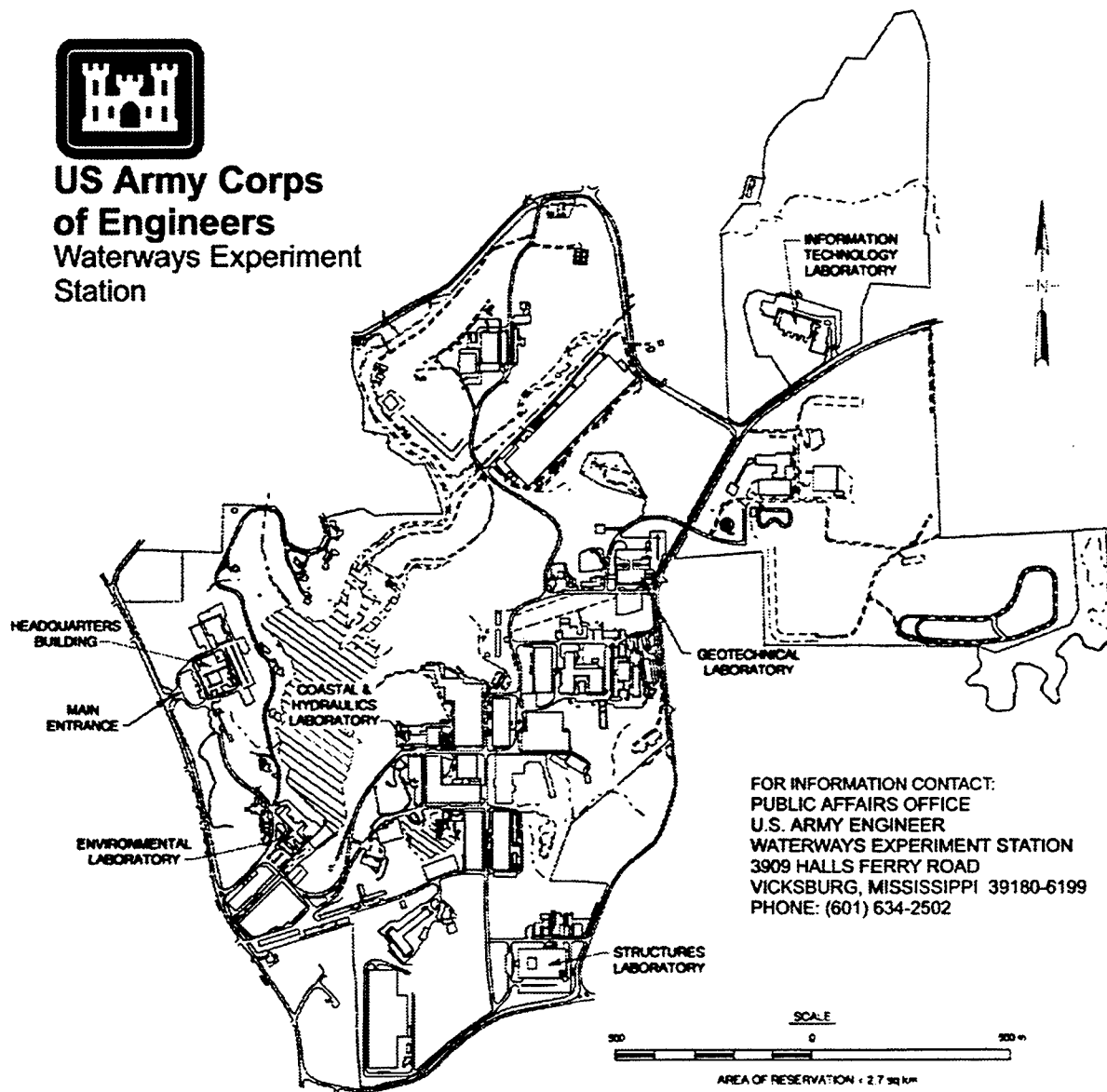
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Final report

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# Preface

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The work reported herein was funded by the Structures Branch, U.S. Army Engineer District (USAED), Vicksburg, and under the Operations and Maintenance (O&M) Reliability Models Research Program. Messrs. C. C. Hamby and Edward Schilling and Ms. Laura Cespedes, USAED, Vicksburg, were the technical monitors during this project. This report was prepared by Mr. Robert C. Patev, Computer-Aided Engineering Division (CAED), Information Technology Laboratory (ITL), Waterways Experiment Station (WES), with contributions by Dr. Mary Ann Leggett, CAED, ITL, and technical oversight by Dr. Reed Mosher, Chief, Structural Mechanics Division, Structures Laboratory, WES. The work was performed under the general supervision of Mr. H. Wayne Jones, Acting Chief, CAED, and Dr. N. Radhakrishnan, Director, ITL.

At the time of publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Robin R. Cababa, EN.

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# 1 Introduction

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A reliability assessment was performed to examine potential modes of unsatisfactory performance during normal operating and project floods for a reinforced concrete box culvert drainage structure and pumping plant. The structure is located under a 10.06-m-high (33-ft) levee section and is composed of reinforced concrete which has suffered severe structural deterioration and exposure of reinforcement. This structure is a critical element in the river levee system that protects a metropolitan area from both river and bayou flooding. Loss of this structure during a project flood event would lead to high consequences for damage because of its proximity to a densely populated area.

The box culvert drainage structure and pumping plant was built in 1935. The existing drainage structure consists of a reinforced concrete box culvert with six openings (two 1.65 m (5 ft, 5 in.)  $\times$  3.38 m (11 ft, 1 in.), two 1.69 m (5 ft, 6 1/2 in.)  $\times$  3.38 m (11 ft, 1 in.), and two 1.73 m (5 ft, 8 in.)  $\times$  3.38 m (11 ft, 1 in.)). The primary function of the structure is to operate as a drainage outlet for the waters and debris from the bayou into the main river system. The structure which is 111.65 m (386 ft) long and 12.49 m (41 ft) wide was built through the levee that protects a metropolitan area from flooding. The cross section of the drainage structure, pumping plant, and levee is shown in Figure 1.

During periods of high water on the river, the six culverts are closed to river flow by sluice gates located on the riverside of the structure. Flows on the riverside can exceed 5,946.54 m<sup>3</sup>/sec (210,000 ft<sup>3</sup>/sec) and 7,220.8 m<sup>3</sup>/sec (255,000 ft<sup>3</sup>/sec) during 1- and 2-percent exceedence events, respectively. For periods of high water in the bayou, the middle four sluice gates are opened on both sides and act as gravity-fed drains while the outer two culverts are pumped by two horizontal axial flow pumps. The outer two culvert gates on the bayou side of the structure always remain closed because the pumps feed from the pump house back into the roofs of the outer culverts. The condition of simultaneously high water in both the bayou and river rarely occurs, since both bodies have different drainage areas.

The box culvert drainage structure has the capacity to handle approximately 59.46 m<sup>3</sup>/sec (2,100 ft<sup>3</sup>/sec) with 0.3048 m (1 ft) of submerged head or 2.54-cm (1-in.) runoff in a 24-hour period. The pumping plant was designed to add an additional capacity of 6.29 m<sup>3</sup>/sec (222 ft<sup>3</sup>/sec). Currently, the actual pump capacity is probably only half that, since the pumps and suction bells are severely corroded and in poor condition. However, the flows in the bayou may sometimes peak in excess of 141.58 m<sup>3</sup>/sec (5,000 ft<sup>3</sup>/sec) over a short duration if



upstream diversion gates cannot be opened. Overtopping of the bayou levees into the metropolitan areas does occur during peak flows events in the bayou.

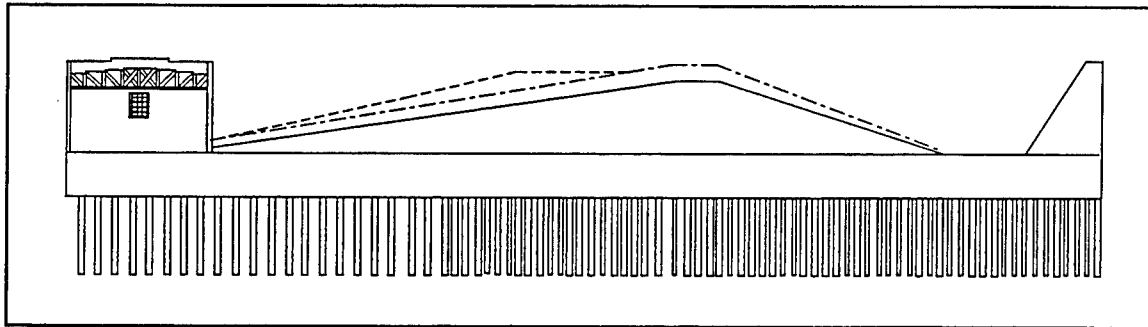


Figure 1. Cross section of drainage structure and pumping plant

The drainage structure and pumping plant have deteriorated greatly primarily due to age and continuous use. During the repairs and dewatering of the structure in 1988, inspection of the inside of the culverts was possible. During this inspection, numerous spalls of concrete and “honeycomb” pockets of exposed reinforcement in the walls of the culvert were discovered. The deterioration of the reinforcement in these exposed areas was so extensive that the reinforcement could be considered completely ineffective. In addition, the design height of the levee was originally specified to be 7.62 m (25 ft) above the top of the structure. Since construction of the structure in 1935, the height of the levee has been increased an additional 2.44 m (8 ft) to its present height of 10.06 m (33 ft) above the top of the structure (shown as dashed lines in Figure 1). This has caused structural cracks (0.953 cm (3/8 in.) to 1.27 cm (1/2 in.)) to develop which ran in both the transverse and longitudinal directions of the culvert roof and walls.

The extent of the degradation of the reinforced concrete experienced during this inspection led to serious questions regarding the safety and the structural integrity of the project. The ability of the structure to perform satisfactorily during a project flood event of any duration coupled with the fact that the structure is adjacent to the hospital facility and in a highly populated downtown area prompted serious concern regarding the structure.

## 2 Deterministic Model

---

### Introduction

The deterministic model developed for the drainage structure has been refined based on the failure and collapse of an exterior wall of the six-barrel culvert structure. The collapse of an exterior wall would allow the levee crown to subside, disrupting the capabilities of the consolidated levee soils and creating a zone where the levee could be breached and flooding could propagate into the city area. A failure of the culvert during a major flood event, i.e., 1-percent exceedence, could cause a large portion of the city to become flooded creating large dollar costs for flood damage, and a large population at risk, especially if the event were to occur with little or no warning.

The model utilizes the behavior of the exterior culvert wall as a simple reinforced concrete beam which is analyzed for its capacity in both moment and shear. This representative beam segment is subjected to lateral earth pressures, internal water pressures, and axial loads from the soil and concrete above. The various loadings on the beam that are used in the model are shown in Figure 2. Figure 3 shows the elevations and dimensions of the exterior culvert. Figure 4 shows the beam, its sectional properties, and the resultant trapezoidal loading.

The deterministic model is simplified by the assumption that longitudinal cracks exist in the top corners of the exterior wall of the culvert. These cracks have been verified from inspection of the culvert during low water times prior to the pooling of lock and dam downstream and from the dewatering and inspection of the culverts in 1988. The interior culvert walls were once considered for a performance mode, but since the force from the lateral earth pressures is much greater than internal water pressures, the exterior walls were considered to be the most crucial elements of the structure.

The deterministic model establishes the limit state as a capacity versus demand relationship in both moment and shear using basic reinforced concrete design and analysis procedures for the beam. This equation is simply expressed for either moment or shear as

$$\text{Limit State} = \frac{\text{Capacity}}{\text{Demand}} \quad (1)$$

## EXTERNAL LOADING ON STRUCTURE

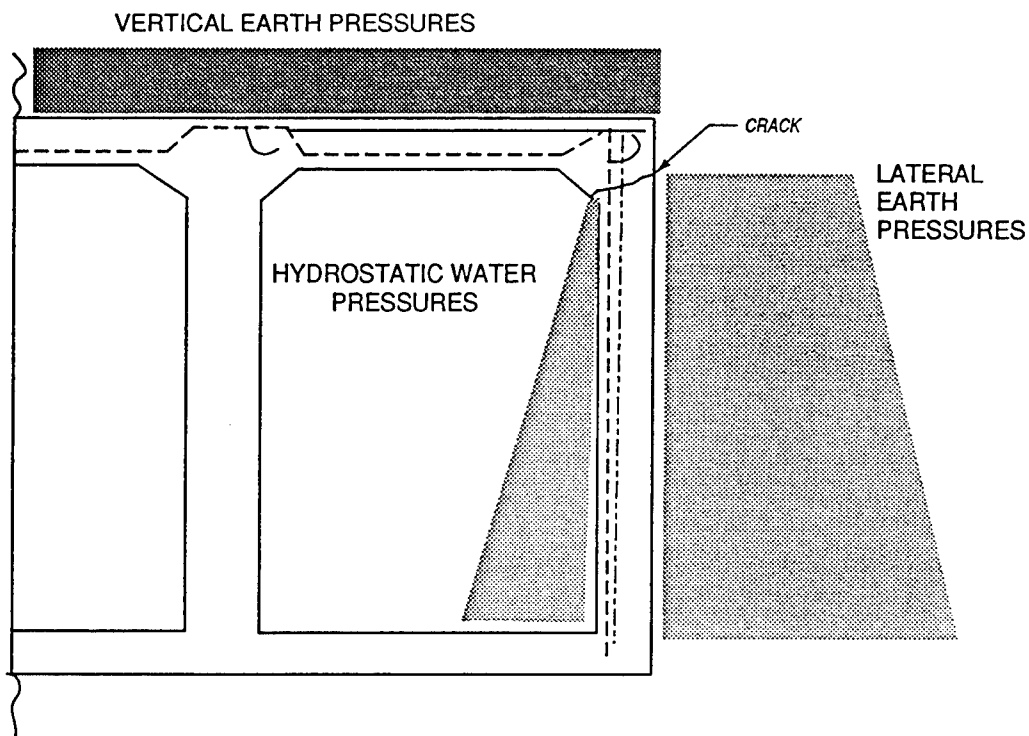


Figure 2. External loadings on culvert walls

Typically, in the design of reinforced concrete, a beam/column is first designed to carry a design moment based on the loads that are applied to the structure. Next, the beam/column is designed to carry the shear from those same applied loads. If the reinforced concrete beam/column is considered to perform unsatisfactorily in moment, the concrete beam/column does not actually collapse, but its moment demand is greater than the moment capacity. The ability of the reinforced concrete beam/column to carry the shear becomes the most crucial factor. If the demand in shear is greater than the capacity of the beam/column, the beam/column will perform unsatisfactorily and the wall will most likely collapse.

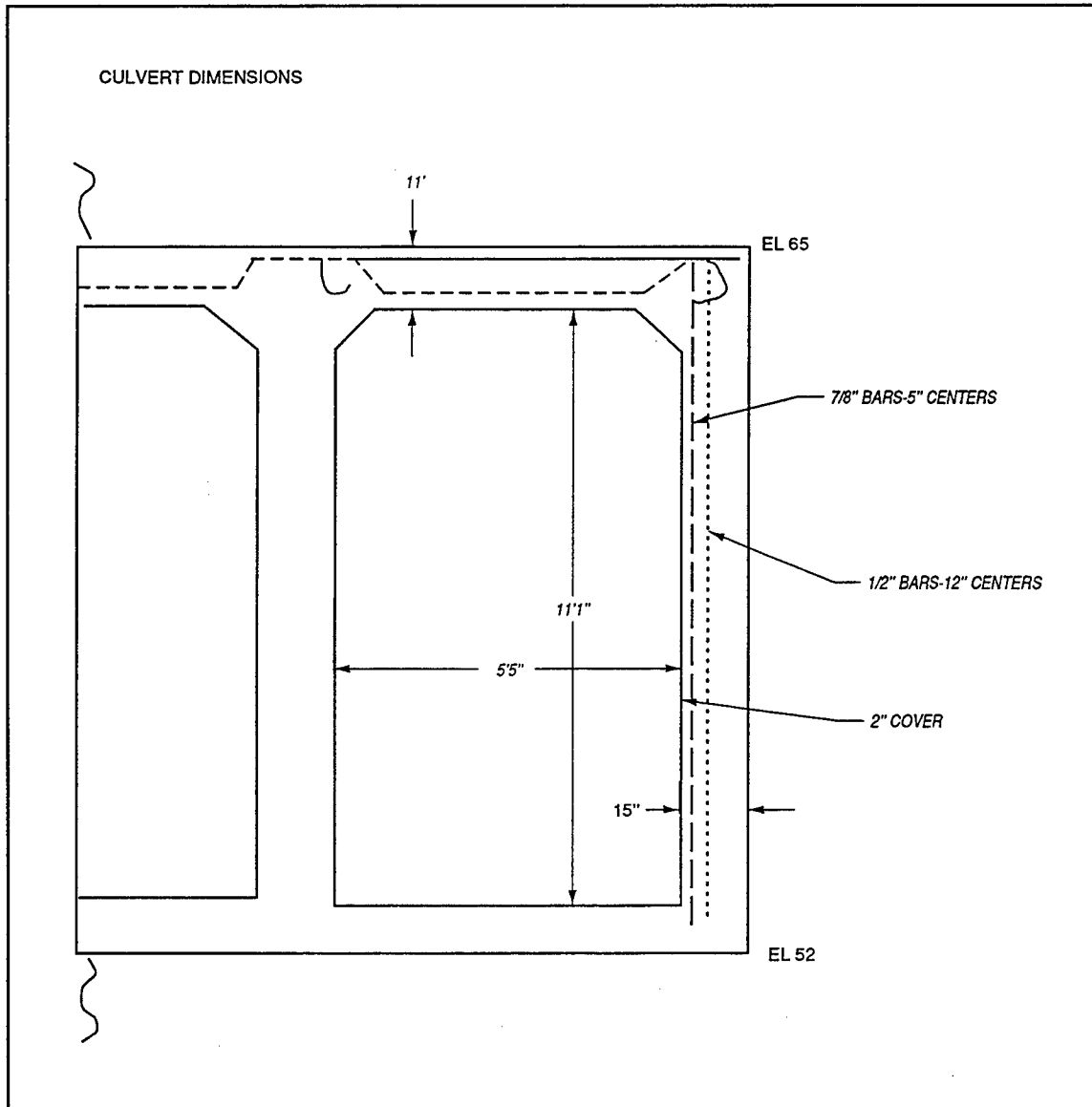


Figure 3. Elevations and dimensions of exterior culvert

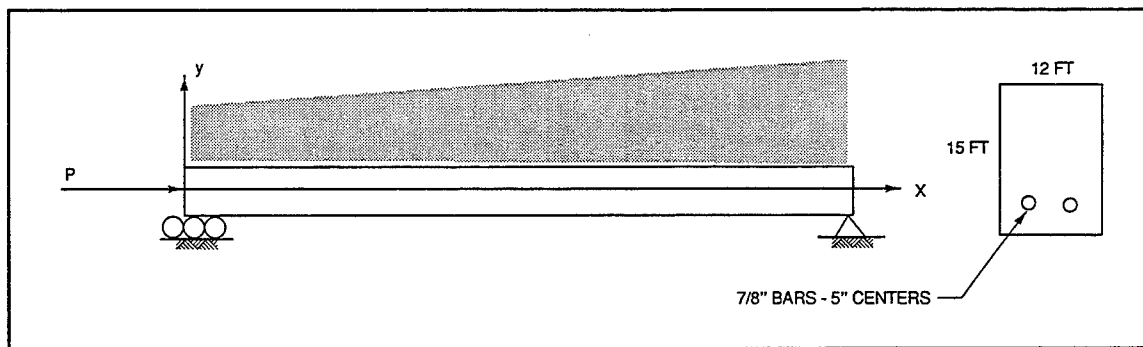


Figure 4. Representative beam model

## Moment Capacity/Demand

The moment capacity for the reinforced concrete beam was determined using equations utilized in the design of reinforced concrete structures. These equations can be derived from reinforced concrete textbooks, e.g., MacGregor (1992), as well as equations from the American Concrete Institute (ACI) 318-89 (ACI 1989). The use of ACI equations allows the examination of the existing structure to be based on current design standards such that an equivalent comparison to a newer or replacement structure can be made. Using ACI equations, the nominal moment is modified by a strength reduction factor,  $\phi$ , to determine the ultimate strength or capacity of the beam. This relationship is expressed by:

$$\begin{aligned} \text{Design Strength} &\geq \text{Required Strength} \\ \text{or} \\ \phi \cdot M_n &\geq M_u \end{aligned} \quad (2)$$

where

$\phi$  = strength reduction factor

$M_n$  = nominal moment

$M_u$  = ultimate moment

This strength reduction factor was determined in accordance with ACI Section 9.3.2.2 for axial load and axial load with flexure. The ACI 318-89 (ACI 1989) code allows the  $\phi$  to be increased linearly from 0.70 to 0.90 as  $\phi P_n$  decreases from  $0.10f'_c A_g$  to zero. The equation for this can be shown as

$$\phi = 0.9 - 0.2 \cdot \frac{P_m}{0.1 \cdot f'_c \cdot h \cdot d} \quad (3)$$

where

$P_m$  = mean axial load (kips)

$f'_c$  = compressive strength of concrete (ksi)

$h$  = height of beam (in.)

$d$  = depth to reinforcement (in.)

The mean value for strength reduction factor for the drainage structure was determined to be 0.83. This was based on a mean axial load of 7348.2 kg (16.2 kips).

The moment capacity for the column is derived utilizing equations from the ACI 318-89 (ACI 1989) and using the Corps of Engineers computer program, CASTR (Hamby and Price 1992), to determine ultimate moment for a singly reinforced concrete beam subjected to axial load. This equation is derived as

$$\phi \cdot M_n = \phi \cdot 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(d - \frac{a}{2}\right) - P_n \cdot \left(d - \frac{h}{2}\right) \quad (4)$$

where

$$a = \frac{A_s \cdot f_y + P_n}{0.85 \cdot f'_c \cdot b}$$

where

$A_s$  = area of steel (in.<sup>2</sup>)

$f_y$  = yield strength of steel (ksi)

$P_n$  = axial load (kips)

$f'_c$  = compressive strength of concrete (ksi)

$b$  = width of section (in.)

and

$d$  = depth to reinforcement (in.)

$h$  = height of section (in.)

$\phi$  = strength reduction factor

$M_n$  = nominal moment

The demand moment was determined from using the combination of load cases from beam tables for a uniform and triangular loading and using the Corps of Engineers, beam-column Soil-Structure Interaction (SSI) computer program, CBEAMC (Dawkins 1994). This equation can be represented as

$$M = \frac{(2w_a + w_b)}{6} \cdot x \cdot l - \frac{w_a \cdot x^2}{2} + \frac{1/2(w_b - w_a) \cdot x^3}{3 \cdot l} \quad (5)$$

$$\frac{dM}{dx} = \frac{(2w_a + w_b) \cdot l}{6} - w_a \cdot x + \frac{1/2(w_b - w_a)}{l} \cdot x^2$$

where

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$w_a$  = distributed load at support  $a$  (lb / in.)

$w_b$  = distributed load at support  $b$  (lb / in.)

$l$  = length from support  $a$  to  $b$  (in.)

where

$$a = \frac{1/2(w_b - w_a)}{l}$$

$$b = -w_a$$

$$c = \frac{(2w_a + w_b) \cdot l}{6}$$

## Shear Capacity/Demand

The shear capacity of the culvert wall was determined by using ACI 318-89 (ACI 1989) Equation 11-4, for shear with axial compression effects. This equation is shown as

$$V_c = 2 * \left( 1 + \frac{Nu}{2000 \cdot A_g} \right) \cdot \sqrt{f'_c} \cdot b_w \cdot d \quad (6)$$

where

$Nu/A_g$  = positive in compression and has unit of psi

$f'_c$  = compressive strength of concrete (ksi)

$b_w$  = width of section (in.)

$d$  = depth of section (in.)

The shear demand was derived by using the combination of load cases from beam tables for both a uniform and triangular loading and using the Corps of Engineers beam-column SSI computer program, CBEAMC (Dawkins 1994). This equation can be derived as

$$V = \frac{w_a \cdot l}{2} + \frac{2(1/2 \cdot l \cdot (w_b - w_a))}{3} \quad (7)$$

where

$w_a$  = distributed load at support  $a$  (lb / in.)

$w_b$  = distributed load at support  $b$  (lb / in.)

$l$  = length from support  $a$  and  $b$  (in.)



## 3 Probabilistic Model

### Constants

The constants used in the model reflect values that were deemed capable of being held as a constant with confidence. Primarily, these constants represented the elevations of the levee and structure and the unit weight of water and are summarized in Table 1.

### Elevation of Top of Levee

The mean profile for the levee was determined from three profiles of the levee recently taken at different stations on the levee at the structure. These stations were Sta. 1254+67.98, Sta. 1253 +67.98, and Sta. 1252+67.98. The mean profile showed an mean elevation at the crown at el 29.87 m (98 ft). The mean profile of the levee is shown in Figure 5.

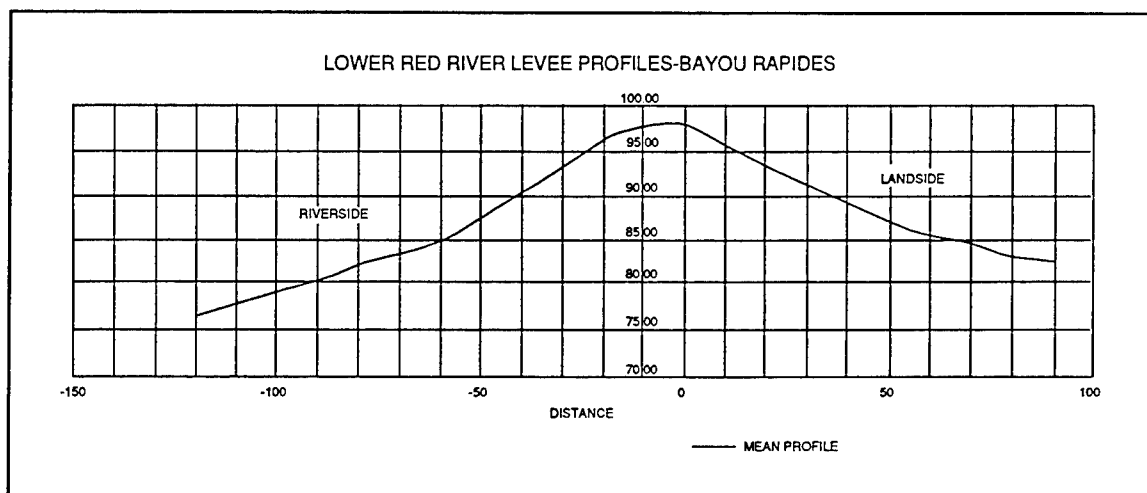


Figure 5. Mean profile of river levee

### Elevations and Interior Height of Culvert

The elevation of the top and the bottom of the culvert was determined to be el 19.81 m (65 ft) and el 15.85 m (52 ft), respectively. The interior height of the

culvert was taken as 3.38m (133 in.). The elevations and height of culvert used as constants in the analysis are shown in Figure 3.

## Unit Weight of Water

A value of 999.55 kg/m<sup>3</sup> (62.4 pcf) was used as the constant for the unit weight of water.

| <b>Table 1</b><br><b>List of Constants in Probabilistic Model</b> |                                     |
|---|-------------------------------------|
| Elevation of top of levee   | 29.87 m (98 ft)                     |
| Elevations - top of culvert                                       | 19.81 m (65 ft)                     |
| Elevations - bottom of culvert                                    | 15.85 m (52 ft)                     |
| Interior height of culvert  | 3.38 m (133 in.)                    |
| Unit weight of water  | 999.55 kg/m <sup>3</sup> (62.4 pcf) |

## Variables

The nine variables, their distributions types, and statistical values for their means,  $\mu$ , and standard deviations,  $\sigma$ , are summarized in Table 2. Each variable has a particular effect on both the capacity and demand side of the limit state equation. Each variable represents some true variability in the modeling of the drainage structure, and this variability will account for the many likely combinations that are possible.

### Elevation of Water - River

**Normal operating.** The values for the elevation of the water on the river during normal operating conditions can range from 19.5 m (64 ft) to 20.42 m (67 ft). A uniform distribution with a mean value of 19.96 m (65.5 ft) and a standard deviation of 0.26 m (0.866 ft) was used in the model.

**2-percent exceedence event.** The values for the elevation of the water on the river during a 2-percent exceedence event can range from 25.42 m (83.4 ft) to 26.03 m (85.4 ft). A uniform distribution with a mean value of 25.73 m (84.4 ft) and a standard deviation of 0.176 m (0.5774 ft) was used in the model.

**1-percent exceedence event.** The values for the elevation of the water on the river during a 1-percent exceedence event can range from 26.55 m (87.1 ft) to 27.16 m (89.1 ft). A uniform distribution with a mean value of 26.86 m (88.1 ft) and a standard deviation of 0.176 m (0.5774 ft) was used in the model.

## **Elevation of Water - Bayou**

**Normal operating.** The values for the elevation of the water on the river during normal operating conditions can range from 19.51 m (64 ft) to 22.25 m (73 ft). A uniform distribution with a mean value of 20.88 m (68.5 ft) and a standard deviation of 0.792 m (2.598 ft) was used in the model.

**Bayou flood event.** A stage flood condition for the bayou was not analyzed for this study since the lowest hydrostatic pressures in the culvert would occur during normal operating events and not during a bayou flood event. A worst case scenario of normal to flood events waters on the river side and normal operating water in the bayou was utilized for this study.

## **Unit Weight of Soil**

The values for the unit weight of soil in the levee were taken from borings in the levee made in 1989. A normal distribution with a mean unit weight of  $1,922.22 \text{ kg/m}^3$  (120 pcf) with a standard deviation of  $80.09 \text{ kg/m}^3$  (5 pcf) was determined.

## **At-rest Earth Pressure Coefficient**

The values for the at-rest earth pressure coefficient were estimated based on research from Brooker and Ireland (1965), Mayne, Jackson, and Kuljhawy (1989), and Mesri (1987). To represent the uncertainty expressed in this variable, a distribution was determined to range from 0.5 to 0.9. A uniform distribution was used with a mean of 0.7 and a standard deviation of 0.1155.

## **Width of Culvert Wall**

The original width of the culvert wall was designed to be 0.381 m (15 in.). During a recent inspection in 1989, the reinforcement in the culvert walls has become exposed and rusted. This indicates that the 0.0508-m (2-in.) cover has eroded over the past 60 years. To model this loss of width, a uniform distribution with a mean of 0.356 m (14 in.) and a standard deviation of 0.015 m (0.5774 in.) was utilized.

## **Unit Weight of Concrete**

The values for the unit weight of concrete in the structure were from tests conducted during dewatering of the structure in 1989. A normal distribution with a mean unit weight of  $2,322.68 \text{ kg/m}^3$  (145 pcf) with a standard deviation of  $80.09 \text{ kg/m}^3$  (5 pcf) was determined.

## Compressive Strength of Concrete

The values for the compressive strength of concrete in the structure from tests conducted during dewatering of the structure in 1989. These values were also confirmed from the specifications for the pumping plant in 1932. A normal distribution with a mean compressive strength of 17,236.9 kPa (2,500 psi) with a standard deviation of 3,447.4 kPa (500 psi) was used.

## Area of Reinforcing Steel

The area of the reinforcing steel is directly dependent upon the amount of cover that was lost. If the entire 5.08 cm (2-in.) cover had been removed by erosion and the steel had been exposed, the area of the reinforcing steel would be reduced. If the cover was not removed, the steel would be intact and the area would still be the original area. In the Monte Carlo simulations, a correlation coefficient of 1 was used to account for this fact.

**Not exposed.** The area of the reinforcing steel that was not exposed was based on two 2.22 cm (7/8-in.) bars per metre (foot) of wall. This yielded an area for the reinforcing steel of 7.74 cm<sup>2</sup> (1.2 in.<sup>2</sup>). Since the type of bar was not specified (round or square), a normal distribution of 7.74 cm<sup>2</sup> (1.2 in.<sup>2</sup>) with a standard deviation of 0.645 cm<sup>2</sup> (0.1 in.<sup>2</sup>) was used.

**Exposed.** To account for the corrosion of the steel that has been exposed in the structure, the area of the reinforcing steel was assumed to have a normal distribution of 7.09 cm<sup>2</sup> (1.1 in.<sup>2</sup>) with a standard deviation of 0.645 cm<sup>2</sup> (0.1 in.<sup>2</sup>). The variation in area could have been much larger, but without actual measurements this range should be sufficient to model loss of area.

## Yield Strength of Reinforcing Steel

The yield strength of the reinforcing steel was based on knowing that in general two different yield strengths of steel, 275,790 kPa (36 ksi) or 248,211 kPa (40 ksi) were being used in the field. Since no information is available from the specifications, a uniform distribution between 275,790 kPa (36 ksi) and 248,211 kPa (40 ksi) was used. The mean value for yield strength of 26,200 kPa (38 ksi) with a standard deviation of 7,928.97 kPa (1.15 ksi) was used in the analysis.

| <b>Table 2</b>  |                     |                                      |                                    |
|---|---------------------|--------------------------------------|------------------------------------|
| <b>List of Random Variables</b>   |                     |                                      |                                    |
| <b>Random Variable</b>  | <b>Distribution</b> | <b><math>\mu</math></b>              | <b><math>\sigma</math></b>         |
| <b>Elevation of water - Riverside</b>   |                     |                                      |                                    |
| Normal operating  | Uniform             | 19.96 m (65.5 ft)                    | 0.26 m (0.866 ft)                  |
| 2-percent exceedence event  | Uniform             | 25.73 m (84.4 ft)                    | 0.176 m (0.5774 ft)                |
| 1-percent exceedence event  | Uniform             | 26.86 m (88.1 ft)                    | 0.176 m (0.5774 ft)                |
| <b>Elevation of water - Bayou side</b>  |                     |                                      |                                    |
| Normal operating  | Uniform             | 20.88 m (68.5 ft)                    | 0.792m (2.598 ft)                  |
| 1%/2% exceedence event  | Uniform             | 20.88 m (68.5 ft)                    | 0.792 m (2.598 ft)                 |
| <b>Bayou flood event (not analyzed for this study)</b>  |                     |                                      |                                    |
| Unit weight of soil   | Normal              | 1,922.22 kg/m <sup>3</sup> (120 pcf) | 80.09 kg/m <sup>3</sup> (5 pcf)    |
| At-rest earth pressure coefficient<br>(Mayne, Jackson, and Kuljhawy 1989, Brooker and Ireland 1965, Mesri 1987) | Uniform             | 0.7                                  | 0.11555                            |
| Width of culvert wall   | Uniform             | 0.356 m (14 in. <sup>2</sup> )       | 0.015 m (0.5774 in. <sup>2</sup> ) |
| Unit weight of concrete   | Normal              | 2,322.68 kg/m <sup>3</sup> (145 pcf) | 80.09 kg/m <sup>3</sup> (5 pcf)    |
| Compressive strength of concrete  | Normal              | 17,236.9 kPa (2,500 psi)             | 3,447.4 kPa (500 psi)              |
| <b>Area of reinforcing steel</b>  |                     |                                      |                                    |
| Not exposed   | Normal              | 3.048 cm (1.2 in. <sup>2</sup> )     | 0.254 cm (0.1 in. <sup>2</sup> )   |
| Exposed   | Normal              | 2.79 cm (1.1 in. <sup>2</sup> )      | 0.254 cm (0.1 in. <sup>2</sup> )   |
| Yield strength of reinforcing steel   | Uniform             | 262,001 kPa (38 ksi)                 | 7,928.97 kPa (1.15 ksi)            |

## 4 Definitions of Reliability Index

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### Introduction

The performance functions or limit state functions introduced above for moment and shear of the reinforced concrete beam can be defined as

$$Z = C - D \quad (8)$$

where

$Z$  = the safety margin

$C$  = capacity

$D$  = demand

Assuming that  $C$  and  $D$  are statistically independent and normally distributed random variables, the mean,  $\mu_Z$ , and variance,  $\sigma_Z^2$ , of  $Z$  can be expressed as

$$\mu_Z = \mu_C - \mu_D \quad (9a)$$

$$\sigma_Z^2 = \mu_C^2 - \mu_D^2 \quad (9b)$$

The event of failure is  $C < D$ , or  $Z < 0$ . The notational probability of unsatisfactory performance,  $p_u$ , is given by

$$p_u = P(Z < 0) = \Phi(-\mu_Z / \sigma_Z) \quad (10)$$

where  $\Phi$  is the cumulative distribution function for a standard normal variate.

The safety index,  $\beta$ , or the reliability index can be defined by the number of standard deviations from the mean of  $Z$  to  $Z = 0$ . The reliability index has been incorporated in many current design codes for steel and concrete structures

target reliability indexes have ranged from a  $\beta$  of 3 to 7 depending upon the criticality of the member or connection being designed. The reliability index,  $\beta$ , is shown in Figure 6 and is commonly expressed as

$$\beta = \mu_Z / \sigma_Z \quad (11)$$

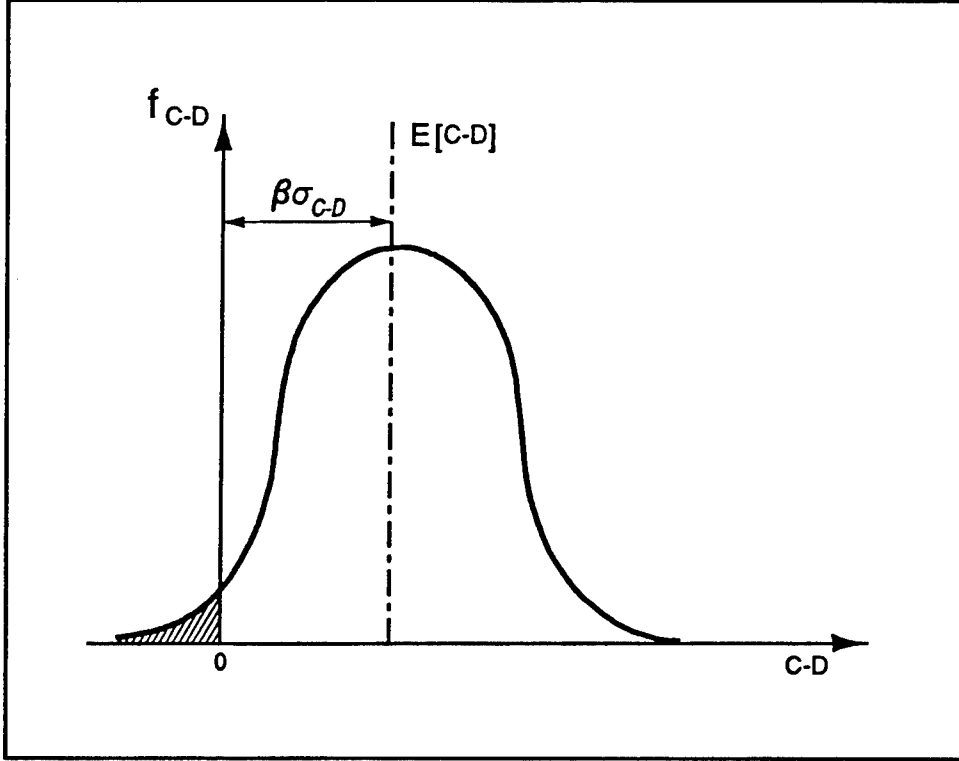


Figure 6. Normal definition of reliability index

An alternative form is to assume that the variables  $C$  and  $D$  are statistically independent lognormal random variables and hence, lognormally distributed as shown in Figure 7. The limit state function,  $Z$ , then becomes a normal random variable and is expressed as

$$Z = \ln (C/D) \quad (12)$$

In addition, the probability of unsatisfactory performance,  $p_u$ , and the reliability,  $R$ , can be related to the reliability index,  $\beta$ , as follows

$$p_u = 1 - \Phi (\beta) \text{ or } p_u = \Phi (-\beta) \quad (13)$$

$$R = 1 - p_u \quad (14)$$

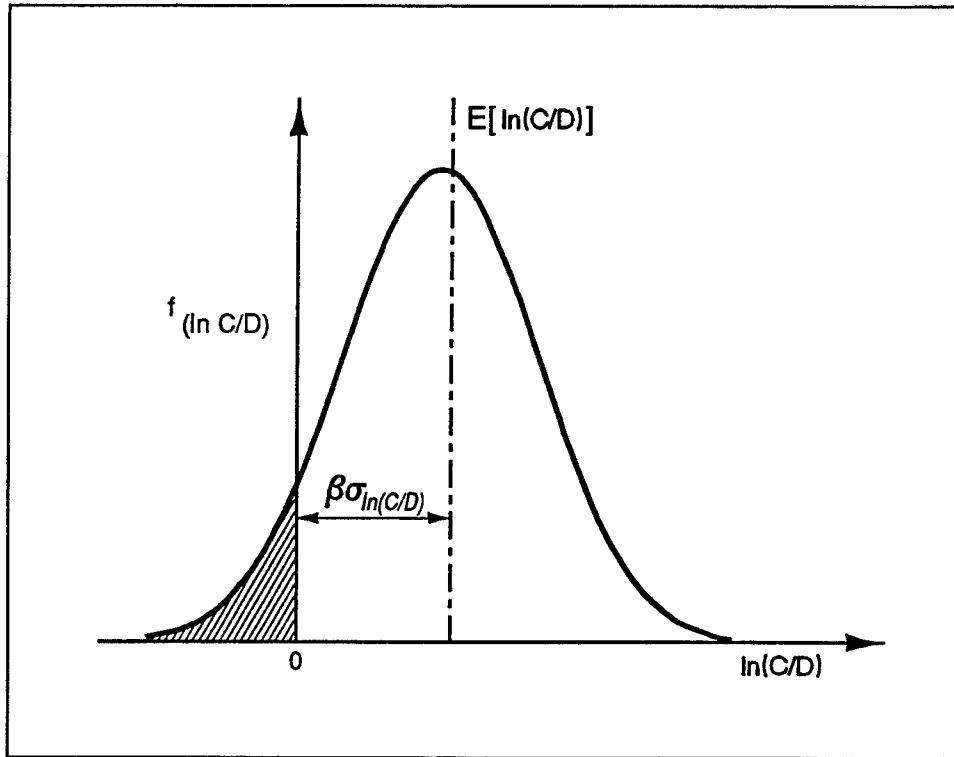


Figure 7. Lognormal definition of reliability index

## First-Order Second Moment (FOSM) - Taylor Series Finite Difference (TSFD) Method

A method used to estimate the reliability index in this report is a First-Order Second Moment-Taylor Series Finite Difference (FOSM-TSFD) Method (Headquarters, Department of the Army 1992). The results for the FOSM-TSFD are shown in Table 3. This reliability method uses lognormal formulations for the reliability index and can be expressed as

$$\beta = \frac{\mu_{\ln F}}{\sigma_{\ln F}} \quad (15)$$

where

$$\mu_{\ln F} = \ln[E[F]] - \frac{\sigma_{\ln F}^2}{2}$$

$$\sigma_{\ln F} = \sqrt{\ln \left[ 1 + \left( \frac{\sigma_F}{E[F]} \right)^2 \right]}$$



$$E[F] \approx F(\mu_i)$$

$$\sigma_F \approx \sqrt{\Sigma \left( \frac{F_i^+ - F_i^-}{2} \right)^2}$$

## Advanced Second Moment

Another FOSM reliability method called the Advanced Second Moment (ASM) is described by Ang and Tang (1984) and Ayyub and Haldar (1984) and was used to check for nonlinearities in the limit state equations for shear and moment. ASM techniques are designed to find the minimum distance or reliability index,  $\beta$ , to the failure surface in multivariable space. This technique is performed using directional cosines, reduced normal variates, and iterations about design points until the minimum value of the reliability index is reached.

The ASM method can be used to assess the reliability of a structure according to a nonlinear performance function that may include nonnormal random variables. Also, the performance function can be in a closed or nonclosed form expression. Implementation of this method requires the use of efficient and accurate numerical algorithms to deal with the nonclosed forms for performance function. The ASM algorithm can be summarized by the following steps:

- a. Assign the mean value for each random variable as a starting design point value, i.e.,  $(X_1^*, X_2^*, \dots, X_n^*) = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)$ .
- b. Compute the standard deviation and mean of the equivalent normal distribution for each nonnormal random variable.
- c. Compute the partial derivative  $\partial Z / \partial X_i$  of the performance function with respect to each random variable evaluated at the design point as needed.
- d. Compute the directional cosine,  $\alpha_i$ , for each random variable at the design point.
- e. Compute the reliability index,  $\beta$ , by satisfying the limit state  $Z = 0$  using a numerical root-finding method.
- f. Compute a new estimate of the design point by using the resulting reliability index  $\beta$  obtained in step e.
- g. Repeat steps b through f until the reliability index,  $\beta$ , converges within an acceptable tolerance,  $\delta$ .

## Monte Carlo Simulations

Monte Carlo simulations (MCS) were run for comparison to the FOSM-TSFD and ASM methods. The results showing the reliability and the probability of unsatisfactory performance values for 20,000 simulations are shown in Table 4. The values for the  $E[C]$  and  $E[D]$  and their deviations are also shown in Figures 8 to 10. The results from the Monte Carlo simulations used in this report use the formulation for reliability expressed from the guidance for Major Rehabilitation Reports (Headquarters, Department of Army 1994). The reliability index,  $\beta$ , for a lognormal definition as shown in Figure 6 can be defined as (Headquarters, Department of the Army 1994).

$$\beta = \frac{\ln\left[\frac{E[C]}{E[D]}\right]}{\sqrt{V_c^2 + V_D^2}} \quad (16)$$

where

$E[C]$  = Expected value of capacity

$E[D]$  = Expected value of demand

$V_c$  = Coefficient of variation of capacity

$V_D$  = Coefficient of variation of demand

(Note: This notation for reliability index should only be used when the  $V_c$  and  $V_D$  are less than 30 percent.)

## 5 Reliability Results

The results showing the reliability index,  $\beta$ , the reliability,  $R$ , and the probability of unsatisfactory performance,  $p_u$ , for the MCS, ASM, and FOSM-TSFD methods for normal operating and 1- and 2-percent exceedence flood events are shown in Tables 3 and 4, respectively. The spreadsheets for the FOSM-TSFD are shown in Figures 8 through 10. The reliability indexes for the MCS and FOSM were calculated using the lognormal definition of  $\beta$  as expressed by Equations 13 and 14.

| <b>Table 3<br/>FOSM-TSFD Results</b> |                           |                       |                         |
|--------------------------------------|---------------------------|-----------------------|-------------------------|
| <b>Limit State 1 - Moment</b>        |                           |                       |                         |
| <b>Flood Event</b>                   | <b><math>\beta</math></b> | <b><math>R</math></b> | <b><math>p_u</math></b> |
| Normal operating                     | -0.2222                   | 0.4121                | 0.5879                  |
| 2-percent exceedence                 | -0.3864                   | 0.3496                | 0.6504                  |
| 1-percent exceedence                 | -0.4503                   | 0.3263                | 0.6737                  |
| <b>Limit State 2 - Shear</b>         |                           |                       |                         |
| <b>Flood Event</b>                   | <b><math>\beta</math></b> | <b><math>R</math></b> | <b><math>p_u</math></b> |
| Normal operating                     | -0.1814                   | 0.4280                | 0.5720                  |
| 2-percent exceedence                 | -0.3328                   | 0.3697                | 0.6303                  |
| 1-percent exceedence                 | -0.3947                   | 0.3465                | 0.6535                  |

**Table 4**  
**ASM and Monte Carlo Simulation Results**

| <b>Limit State 1 - Moment</b> |         |          |                      |
|-------------------------------|---------|----------|----------------------|
| <b>Flood Event</b>            | $\beta$ | <b>R</b> | <b>P<sub>L</sub></b> |
| Normal operating              | -0.1382 | 0.4450   | 0.5550               |
| 2-percent exceedence          | -0.3665 | 0.3570   | 0.6430               |
| 1-percent exceedence          | -0.4584 | 0.3233   | 0.6767               |
| <b>Limit State 2- Shear</b>   |         |          |                      |
| <b>Flood Event</b>            | $\beta$ | <b>R</b> | <b>P<sub>L</sub></b> |
| Normal operating              | -0.0893 | 0.4644   | 0.5356               |
| 2-percent exceedence          | -0.2935 | 0.3846   | 0.6154               |
| 1-percent exceedence          | -0.3785 | 0.3525   | 0.6475               |

|  |      |       |
|--|------|-------|
| Elevation of water- Red River (ft)                         | 65.5 | 0.866 |
| Elevation of water - Bayou Rapides (ft)                    | 66.5 | 2.508 |
| Unit weight of soil (pcf)                                  | 120  | 5     |
| Earth pressure coefficient                                 | 0.7  | 0.116 |
| Width of culvert wall (inches)                             | 14   | 0.577 |
| Unit weight of concrete (pcf)                              | 145  | 5     |
| Compressive strength of concrete (psi)                     | 2500 | 500   |
| Area of reinforcing steel (not exposed) (in <sup>2</sup> ) | 1.2  | 0.1   |
| Area of reinforcing steel (exposed) (in <sup>2</sup> )     | 1.1  | 0.1   |
| Yield strength of steel (ksi)                              | 38   | 1.15  |

|         | 0.01  | 0.02  | 0.04   | 0.17   | 0.06   | 0.00   | 0.02  | 0.02   | 0.00   | SRSS   |
|---------|-------|-------|--------|--------|--------|--------|-------|--------|--------|--------|
| SD-FD 1 | 3.22% | 9.54% | 20.82% | 84.53% | 31.60% | 32.19% | 0.00% | 12.32% | 11.53% | 100.0% |

|         | 0.01  | 0.02  | 0.05   | 0.17   | 0.00  | 0.00  | 0.10   | 0.00  | 0.00  | SRSS   |
|---------|-------|-------|--------|--------|-------|-------|--------|-------|-------|--------|
| SD-FD 2 | 2.95% | 9.15% | 22.82% | 83.42% | 0.00% | 0.00% | 49.28% | 0.00% | 0.00% | 100.0% |

## Chapter 5 Reliability Results

Taylor Series Reliability CalculationsETL ApproachLimit State 1 - Moment      Limit State 2 - Shear

|                    |         |                    |         |
|--------------------|---------|--------------------|---------|
| $\mu_{cnd} =$      | 0.8759  | $\mu_{cnd} =$      | 0.8839  |
| $\sigma_{cnd} =$   | 0.1967  | $\sigma_{cnd} =$   | 0.2007  |
| $\sigma_{incnd} =$ | 0.1996  | $\sigma_{incnd} =$ | 0.2019  |
| $\mu_{incnd} =$    | -0.0443 | $\mu_{incnd} =$    | -0.0368 |
| $\beta =$          | -0.2222 | $\beta =$          | -0.1814 |

|          |        |          |        |
|----------|--------|----------|--------|
| $t$      | 0.9312 | $z$      | 0.3892 |
| $R =$    | 0.412  | $R =$    | 0.4280 |
| $P(U) =$ | 0.5879 | $P(U) =$ | 0.5720 |

Monte Carlo Simulation Results (20,000 iterations)Limit State 1 - Moment

|                    |         |                    |         |
|--------------------|---------|--------------------|---------|
| $E(C) =$           | 43.4921 | $E(C) =$           | 16.3226 |
| $\sigma(C) =$      | 2.1066  | $\sigma(C) =$      | 0.8825  |
| $E(D) =$           | 44.5580 | $E(D) =$           | 18.3822 |
| $\sigma(D) =$      | 7.4990  | $\sigma(D) =$      | 2.7896  |
| $\mu_{incnd} =$    | -0.0242 | $\mu_{incnd} =$    | -0.0158 |
| $\sigma_{incnd} =$ | 0.1751  | $\sigma_{incnd} =$ | 0.1767  |
| $\beta =$          | -0.1382 | $\beta =$          | -0.0893 |

|          |        |          |        |
|----------|--------|----------|--------|
| $t$      | 0.9560 | $z$      | 0.3951 |
| $R =$    | 0.4450 | $R =$    | 0.4644 |
| $P(U) =$ | 0.5550 | $P(U) =$ | 0.5356 |

Limit State 2 - Shear

|          |        |          |        |
|----------|--------|----------|--------|
| $t$      | 0.9712 | $z$      | 0.3974 |
| $R =$    | 0.4644 | $R =$    | 0.4644 |
| $P(U) =$ | 0.5356 | $P(U) =$ | 0.5356 |

Figure 8. (Concluded)

# Reliability Analysis of Bayou Rapides Drainage Structure 2-PERCENT EXCEEDENCE EVENT - AVERAGE WATER BAYOU

|   |      |       |
|---|------|-------|
| Elevation of water- Red River (ft)                        | 84.4 | 0.577 |
| Elevation of water - Bayou Rapides (ft)                   | 68.5 | 0.866 |
| Unit weight of soil (pcf)                                 | 120  | 5     |
| Earth pressure coefficient                                | 0.7  | 0.116 |
| Width of culvert wall (inches)                            | 14   | 0.577 |
| Unit weight of concrete (pcf)                             | 145  | 5     |
| Compressive strength of concrete (psi)                    | 2500 | 500   |
| Area of reinforcing steel (not exposed)(in <sup>2</sup> ) | 1.2  | 0.1   |
| Area of reinforcing steel (exposed) (in <sup>2</sup> )    | 1.1  | 0.1   |
| Yield strength of steel (ksi)                             | 38   | 1.15  |

| Case | Elev.  | Elev.  | $\gamma_{soil}$ | $K_s$ | W1/W2  | $\gamma_{concrete}$ | $f'_c$ | $A_s$ | $f_y$ | $M_{ult}$ | Limit | Shear | Limit |
|------|--------|--------|-----------------|-------|--------|---------------------|--------|-------|-------|-----------|-------|-------|-------|
| Mean | 84.4   | 68.5   | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 48.01 | 16.35 | 17.08 |
| 1    | 84.977 | 68.5   | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 48.09 | 16.35 | 17.11 |
| 2    | 83.823 | 68.5   | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 45.92 | 16.35 | 17.05 |
| 3    | 84.4   | 68.366 | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 45.31 | 16.35 | 16.82 |
| 4    | 84.4   | 67.634 | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 46.71 | 16.35 | 17.33 |
| 5    | 84.4   | 68.5   | 125             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.88     | 48.12 | 16.38 | 17.88 |
| 6    | 84.4   | 68.5   | 115             | 0.7   | 14     | 145                 | 2500   | 1.2   | 38    | 43.43     | 43.9  | 16.32 | 16.28 |
| 7    | 84.4   | 68.5   | 120             | 0.816 | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 52.38 | 16.35 | 19.46 |
| 8    | 84.4   | 68.5   | 120             | 0.584 | 14     | 145                 | 2500   | 1.2   | 38    | 43.66     | 39.63 | 16.35 | 14.7  |
| 9    | 84.4   | 68.5   | 120             | 0.7   | 14.577 | 145                 | 2500   | 1.3   | 38    | 46.73     | 45.99 | 15.79 | 17.08 |
| 10   | 84.4   | 68.5   | 120             | 0.7   | 14.577 | 145                 | 2500   | 1.1   | 38    | 41.17     | 45.99 | 15.79 | 17.08 |
| 11   | 84.4   | 68.5   | 120             | 0.7   | 13.423 | 145                 | 2500   | 1.2   | 38    | 43.33     | 46.03 | 16.92 | 17.08 |
| 12   | 84.4   | 68.5   | 120             | 0.7   | 13.423 | 145                 | 2500   | 1     | 38    | 37.66     | 46.03 | 16.92 | 17.08 |
| 13   | 84.4   | 68.5   | 120             | 0.7   | 14     | 150                 | 2500   | 1.2   | 38    | 43.66     | 46    | 16.35 | 17.08 |
| 14   | 84.4   | 68.5   | 120             | 0.7   | 14     | 140                 | 2500   | 1.2   | 38    | 43.66     | 46.02 | 16.35 | 17.08 |
| 15   | 84.4   | 68.5   | 120             | 0.7   | 14     | 145                 | 3000   | 1.2   | 38    | 44.5      | 45.99 | 17.91 | 17.08 |
| 16   | 84.4   | 68.5   | 120             | 0.7   | 14     | 145                 | 2000   | 1.2   | 38    | 42.36     | 46.03 | 14.63 | 17.08 |
| 17   | 84.4   | 68.5   | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 39.15 | 44.68     | 46.01 | 16.35 | 17.08 |
| 18   | 84.4   | 68.5   | 120             | 0.7   | 14     | 145                 | 2500   | 1.2   | 36.85 | 42.63     | 46.01 | 16.35 | 17.08 |

|         |       |       |        |        |        |        |       |        |        |       |
|---------|-------|-------|--------|--------|--------|--------|-------|--------|--------|-------|
| SD-FD 1 | 0.00  | 0.01  | 0.04   | 0.13   | 0.06   | 0.08   | 0.00  | 0.02   | 0.02   | 0.00  |
|         | 1.04% | 8.60% | 23.05% | 79.87% | 36.01% | 38.69% | 0.12% | 14.10% | 13.14% | 0.00% |

SRSS 100.0%

|         |       |       |        |        |       |       |       |        |       |       |
|---------|-------|-------|--------|--------|-------|-------|-------|--------|-------|-------|
| SD-FD 2 | 0.00  | 0.01  | 0.04   | 0.14   | 0.00  | 0.00  | 0.00  | 0.10   | 0.00  | 0.00  |
|         | 0.97% | 8.28% | 25.07% | 76.74% | 0.00% | 0.00% | 0.00% | 55.69% | 0.00% | 0.00% |

SRSS 100.0%

Figure 9. FOSM-TSFD - 2-percent exceedence event (Continued)

Taylor Series Reliability CalculationsETL ApproachLimit State 1 - Moment      Limit State 2 - Shear

|                   |         |                   |         |
|-------------------|---------|-------------------|---------|
| $\mu_{CD} =$      | 0.9489  | $\mu_{CD} =$      | 0.9574  |
| $\sigma_{CD} =$   | 0.1679  | $\sigma_{CD} =$   | 0.1728  |
| $\sigma_{InCD} =$ | 0.1755  | $\sigma_{InCD} =$ | 0.1790  |
| $\mu_{InCD} =$    | -0.0878 | $\mu_{InCD} =$    | -0.0598 |
| $\beta =$         | -0.3864 | $\beta =$         | -0.3328 |

|     |        |     |        |
|-----|--------|-----|--------|
| $t$ | 0.8861 | $z$ | 0.3702 |
| $t$ | 0.9003 | $z$ | 0.3775 |

|          |        |          |        |
|----------|--------|----------|--------|
| $R =$    | 0.3498 | $R =$    | 0.3687 |
| $P(U) =$ | 0.6504 | $P(U) =$ | 0.6303 |

Monte Carlo Simulation Results (20,000 Iterations)Limit State 1 - Moment

|                   |         |                   |         |
|-------------------|---------|-------------------|---------|
| $E(C) =$          | 43.4651 | $E(C) =$          | 16.3277 |
| $\sigma(C) =$     | 2.1232  | $\sigma(C) =$     | 0.8889  |
| $E(D) =$          | 45.9913 | $E(D) =$          | 17.0941 |
| $\sigma(D) =$     | 6.7238  | $\sigma(D) =$     | 2.8041  |
| $\mu_{InCD} =$    | -0.0565 | $\mu_{InCD} =$    | -0.0459 |
| $\sigma_{InCD} =$ | 0.1541  | $\sigma_{InCD} =$ | 0.1563  |
| $\beta =$         | -0.3685 | $\beta =$         | -0.2935 |

|     |        |     |        |
|-----|--------|-----|--------|
| $t$ | 0.8913 | $z$ | 0.3730 |
| $t$ | 0.9110 | $z$ | 0.3821 |

|          |        |          |        |
|----------|--------|----------|--------|
| $R =$    | 0.3570 | $R =$    | 0.3846 |
| $P(U) =$ | 0.6430 | $P(U) =$ | 0.6154 |

Limit State 2 - Shear

|                   |         |                   |         |
|-------------------|---------|-------------------|---------|
| $E(C) =$          | 16.3277 | $E(C) =$          | 16.3277 |
| $\sigma(C) =$     | 0.8889  | $\sigma(C) =$     | 0.8889  |
| $E(D) =$          | 17.0941 | $E(D) =$          | 17.0941 |
| $\sigma(D) =$     | 2.8041  | $\sigma(D) =$     | 2.8041  |
| $\mu_{InCD} =$    | -0.0459 | $\mu_{InCD} =$    | -0.0459 |
| $\sigma_{InCD} =$ | 0.1563  | $\sigma_{InCD} =$ | 0.1563  |
| $\beta =$         | -0.2935 | $\beta =$         | -0.2935 |

|     |        |     |        |
|-----|--------|-----|--------|
| $t$ | 0.9110 | $z$ | 0.3821 |
| $t$ | 0.9110 | $z$ | 0.3821 |

|          |        |          |        |
|----------|--------|----------|--------|
| $R =$    | 0.3846 | $R =$    | 0.3846 |
| $P(U) =$ | 0.6154 | $P(U) =$ | 0.6154 |

Figure 9. (Concluded)



# Reliability Analysis of Bayou Rapides Drainage Structure 1-PERCENT EXCEEDENCE EVENT - AVERAGE WATER BAYOU

|   |      |       |
|---|------|-------|
| Elevation of water- Red River (ft)                        | 88.1 | 0.577 |
| Elevation of water - Bayou Rapides (ft)                   | 88.5 | 2.598 |
| Unit weight of soil (pcf)                                 | 120  | 5     |
| Earth pressure coefficient                                | 0.7  | 0.116 |
| Width of culvert wall (inches)                            | 14   | 0.577 |
| Unit weight of concrete (pcf)                             | 145  | 5     |
| Compressive strength of concrete (psi)                    | 2500 | 500   |
| Area of reinforcing steel (not exposed)(in <sup>2</sup> ) | 1.2  | 0.1   |
| Area of reinforcing steel (exposed) (in <sup>2</sup> )    | 1.1  | 0.1   |
| Yield strength of steel (ksi)                             | 38   | 1.15  |

| Case | Elev.     | Elev.      | γ soil | K <sub>s</sub> | W1 /W2 | γ concrete | L <sub>s</sub> | A <sub>s</sub> | I <sub>y</sub> | M <sub>ult</sub><br>CASTR CBEAMC | Limit<br>State 1 | Limit<br>State 2 | Shear | Shear | Limit |
|------|-----------|------------|--------|----------------|--------|------------|----------------|----------------|----------------|----------------------------------|------------------|------------------|-------|-------|-------|
| Mean | Red River | Bayou Rap. |        |                |        |            |                |                |                |                                  |                  |                  |       |       |       |
| 1    | 88.677    | 88.5       | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 46.53            | 0.94             | 16.35 | 17.27 | 0.95  |
| 2    | 87.523    | 88.5       | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 46.61            | 0.94             | 16.35 | 17.3  | 0.95  |
| 3    | 88.1      | 71.098     | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 46.45            | 0.94             | 16.35 | 17.24 | 0.95  |
| 4    | 88.1      | 65.902     | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 44.44            | 0.88             | 16.35 | 18.51 | 0.99  |
| 5    | 88.1      | 68.5       | 125    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 48.64            | 0.90             | 16.35 | 18.03 | 0.91  |
| 6    | 88.1      | 68.5       | 115    | 0.7            | 14     | 145        | 2500           | 1.2            | 38             | 43.88                            | 48.64            | 0.90             | 16.38 | 18.06 | 0.91  |
| 7    | 88.1      | 68.5       | 120    | 0.816          | 14     | 145        | 2500           | 1.2            | 38             | 43.43                            | 44.43            | 0.98             | 16.32 | 18.47 | 0.99  |
| 8    | 88.1      | 68.5       | 120    | 0.584          | 14     | 145        | 2500           | 1.2            | 38             | 43.66                            | 52.71            | 0.83             | 16.35 | 19.57 | 0.84  |
| 9    | 88.1      | 68.5       | 120    | 0.7            | 14.577 | 145        | 2500           | 1.3            | 38             | 43.66                            | 40.36            | 1.08             | 16.35 | 14.96 | 1.09  |
| 10   | 88.1      | 68.5       | 120    | 0.7            | 13.423 | 145        | 2500           | 1.2            | 38             | 43.33                            | 46.51            | 0.93             | 15.79 | 17.27 | 0.91  |
| 11   | 88.1      | 68.5       | 120    | 0.7            | 13.423 | 145        | 2500           | 1.2            | 38             | 43.33                            | 46.56            | 0.93             | 16.92 | 17.27 | 0.98  |
| 12   | 88.1      | 68.5       | 120    | 0.7            | 13.423 | 145        | 2500           | 1              | 38             | 37.66                            | 46.56            | 0.81             | 16.92 | 17.27 | 0.98  |
| 13   | 88.1      | 68.5       | 120    | 0.7            | 14     | 150        | 2500           | 1.2            | 38             | 43.66                            | 46.52            | 0.94             | 16.35 | 17.27 | 0.95  |
| 14   | 88.1      | 68.5       | 120    | 0.7            | 14     | 140        | 2500           | 1.2            | 38             | 43.66                            | 46.54            | 0.94             | 16.35 | 17.27 | 0.95  |
| 15   | 88.1      | 68.5       | 120    | 0.7            | 14     | 145        | 3000           | 1.2            | 38             | 44.5                             | 48.52            | 0.86             | 17.91 | 17.27 | 1.04  |
| 16   | 88.1      | 68.5       | 120    | 0.7            | 14     | 145        | 2000           | 1.2            | 38             | 42.36                            | 46.55            | 0.91             | 14.63 | 17.27 | 0.85  |
| 17   | 88.1      | 68.5       | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38.15          | 44.68                            | 48.53            | 0.98             | 16.35 | 17.27 | 0.95  |
| 18   | 88.1      | 68.5       | 120    | 0.7            | 14     | 145        | 2500           | 1.2            | 38.85          | 42.63                            | 46.53            | 0.92             | 16.35 | 17.27 | 0.95  |

|         |       |        |        |        |        |        |       |        |        |       |      |        |
|---------|-------|--------|--------|--------|--------|--------|-------|--------|--------|-------|------|--------|
| SD-FD 1 | 0.00  | 0.04   | 0.04   | 0.13   | 0.06   | 0.06   | 0.02  | 0.02   | 0.02   | 0.00  | SRSS | 100.0% |
|         | 0.97% | 25.54% | 22.69% | 76.31% | 35.99% | 36.66% | 0.12% | 14.03% | 13.14% | 0.00% |      |        |

|         |       |        |        |        |       |       |        |       |       |       |      |        |
|---------|-------|--------|--------|--------|-------|-------|--------|-------|-------|-------|------|--------|
| SD-FD 2 | 0.00  | 0.04   | 0.04   | 0.13   | 0.00  | 0.00  | 0.10   | 0.00  | 0.00  | 0.00  | SRSS | 100.0% |
|         | 0.96% | 24.45% | 24.65% | 75.41% | 0.00% | 0.00% | 55.74% | 0.00% | 0.00% | 0.00% |      |        |

Figure 10. FOSM-TSFD - 1-percent exceedence event

# Taylor Series Reliability Calculations

## ETL Approach

### Limit State 1 - Moment      Limit State 2 - Shear

|                    |         |                    |         |
|--------------------|---------|--------------------|---------|
| $\mu_{C/D} =$      | 0.3383  | $\mu_{C/D} =$      | 0.9469  |
| $\sigma_{C/D} =$   | 0.1661  | $\sigma_{C/D} =$   | 0.1707  |
| $\sigma_{Inc/D} =$ | 0.1756  | $\sigma_{Inc/D} =$ | 0.1789  |
| $\mu_{Inc/D} =$    | -0.0791 | $\mu_{Inc/D} =$    | -0.0709 |
| $\beta =$          | 0.4503  | $\beta =$          | 0.3947  |

|          |        |     |        |
|----------|--------|-----|--------|
| $t$      | 0.8697 | $z$ | 0.3605 |
| $R =$    | 0.3283 | $t$ | 0.8839 |
| $P(U) =$ | 0.6737 | $z$ | 0.3690 |

|          |        |
|----------|--------|
| $R =$    | 0.3465 |
| $P(U) =$ | 0.6535 |

## Monte Carlo Simulation Results (20,000 Iterations)

### Limit State 1 - Moment

|                    |          |
|--------------------|----------|
| $E(C) =$           | 43.44789 |
| $\sigma(C) =$      | 2.1130   |
| $E(D) =$           | 46.5169  |
| $\sigma(D) =$      | 6.5458   |
| $\mu_{Inc/D} =$    | -0.0683  |
| $\sigma_{Inc/D} =$ | 0.1489   |
| $\beta =$          | -0.4584  |

|          |        |     |        |
|----------|--------|-----|--------|
| $t$      | 0.8677 | $z$ | 0.3591 |
| $R =$    | 0.3233 | $t$ | 0.8882 |
| $P(U) =$ | 0.6767 | $z$ | 0.3714 |

|          |        |
|----------|--------|
| $R =$    | 0.3526 |
| $P(U) =$ | 0.6475 |

### Limit State 2 - Shear

|                    |         |
|--------------------|---------|
| $E(C) =$           | 16.3234 |
| $\sigma(C) =$      | 0.8848  |
| $E(D) =$           | 17.2848 |
| $\sigma(D) =$      | 2.4394  |
| $\mu_{Inc/D} =$    | -0.0572 |
| $\sigma_{Inc/D} =$ | 0.1512  |
| $\beta =$          | -0.3785 |

Figure 10. (Concluded)

## 6 Conclusions

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The results from the FOSM-TSFD, ASM, and MCS indicate a good comparison in the reliability index estimation. The main focus from the results is that all the reliability procedures indicate a very low reliability index (i.e., negative) or a high probability of unsatisfactory performance for the structure under both normal operating and the 1- and 2-percent exceedence flood events. These values for unsatisfactory performance would indicate that the exterior walls of the culvert should have already performed unsatisfactorily and collapsed during a flood event. However, the structure is still operative because a 1- or 2-percent exceedence event has not occurred since the structure was built in 1935. Similar structures have performed unsatisfactorily during high water events. An example of the collapse of a box culvert drainage structure in a rural area upstream of the example structure is shown in Figure 11.

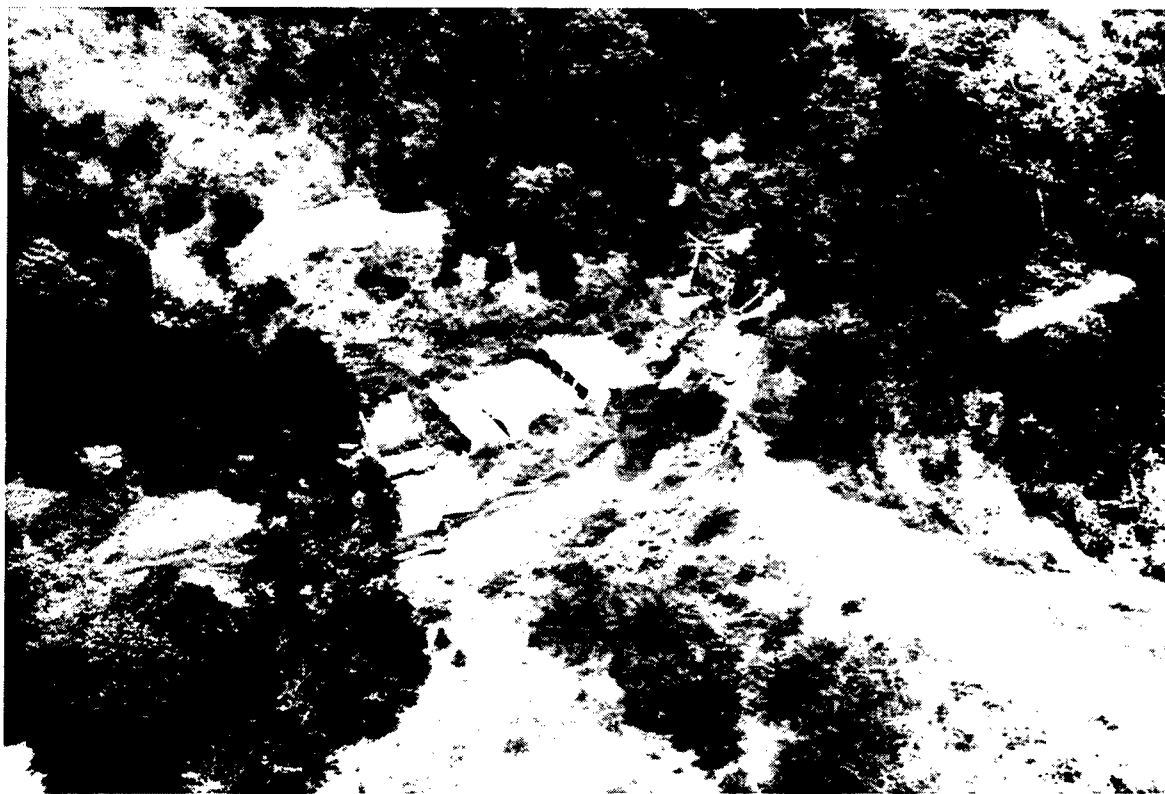


Figure 11. Unsatisfactory performance of a box culvert structure

The expected performance of this drainage structure during these flood events can also be illustrated by examining the moment capacity of the structure under a 1-percent exceedence flood event. The probability of unsatisfactory performance,  $p_u$ , in moment was determined from the reliability analysis to be 0.6767. This number indicates that out of 1,000 structures of similar like and condition, approximately 677 structures should perform unsatisfactorily in moment. This leaves 323 similar structures that would perform satisfactory in their moment capacity. Hence, there is still the possibility that the drainage structure would not collapse during a 1-percent flood event because it has either not failed in moment (most unlikely scenario), or has failed in moment and not in shear (most likely scenario).

The joint probability of unsatisfactory performance in both moment and shear can also be determined. Since the probability of unsatisfactory performance,  $p_u$ , in shear was 0.6475, the joint probability would be  $0.4382(Z)$  or  $438.2(Z)$  structures out of 1,000, where  $Z$  is the respective probability of a 1-percent exceedence event. The joint probabilities for the two failure modes (moment and shear) and for normal operating and two flood events, 1- and 2-percent exceedence are shown in the event tree in Figure 12.

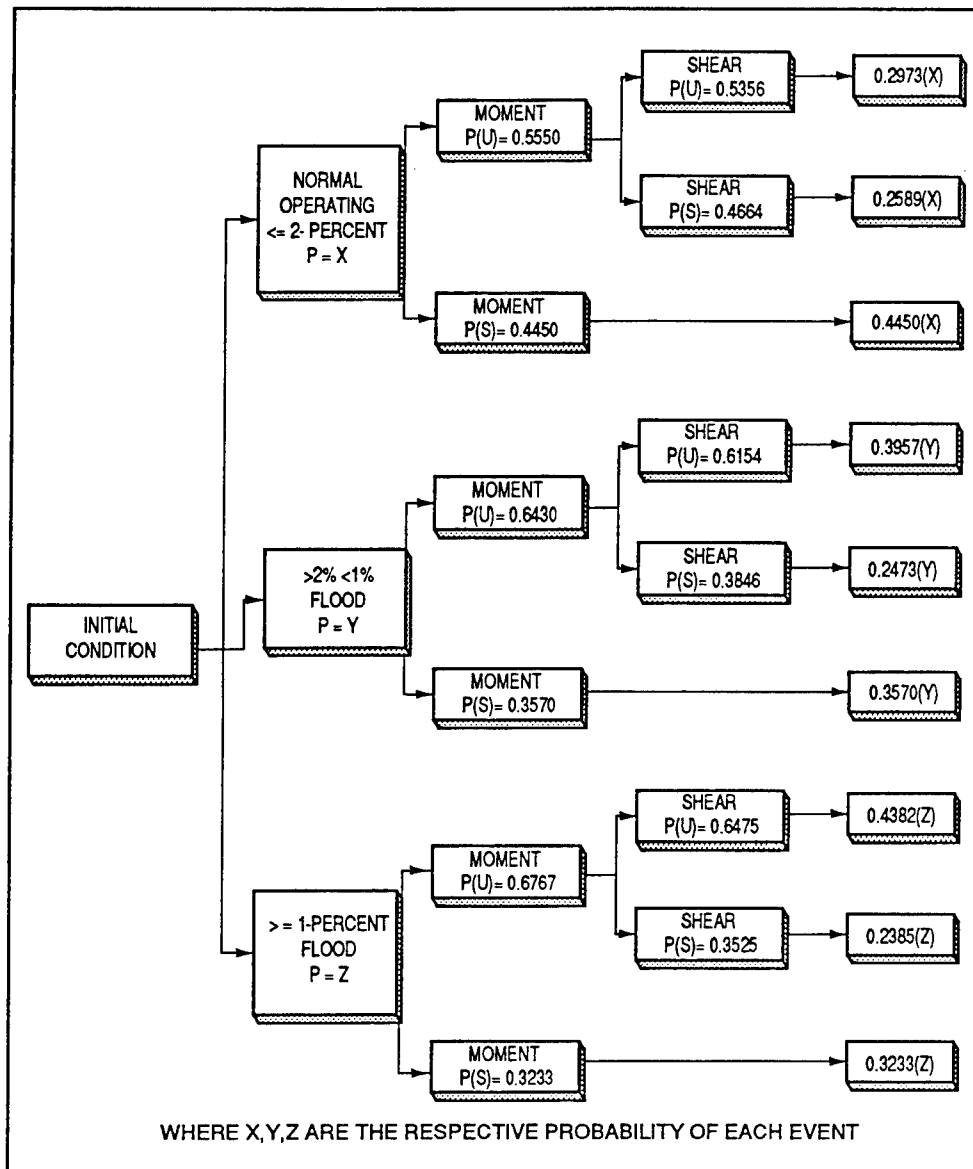


Figure 12. Event Tree for joint probabilities from the Monte Carlo Simulation

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| <b>13. ABSTRACT (Maximum 200 words)</b><br><p>A reliability assessment was performed to examine potential models of unsatisfactory performance of six-barrel drainage structure and pumping plant. The structure is located under a 33-ft high levee section and is composed of reinforced concrete which has suffered severe structural deterioration and exposure of reinforcement. This structure is a critical element in the levee system that protects a major metropolitan area from river and bayou flooding. Loss of this structure during a project flood event would lead to a high probability of loss of life because of its proximity to a highly populated downtown area. Using soil-structure interaction analyses, the outer culvert wall was modeled as a beam-column subjected to lateral and vertical earth pressures, and hydrostatic water pressures. Monte Carlo Simulations, Advanced Second Moment, and First-Order Second Moment techniques were utilized to examine the reliability of the culvert in the limit states of ultimate shear and moment for normal operating conditions and for project floods of 1 and 2 percent occurrence.</p> |   |  |   |  |
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